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Lower and Upper Approximation of Fuzzy Ideals in a Semiring

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Abstract— In this paper, we introduce the rough fuzzy ideals of a semiring. We also introduce and study rough fuzzy prime ideals of a semiring.

Index Terms— Semiring, lower approximation, upper approximation, fuzzy ideal, fuzzy prime ideal, rough ideal.

1 INTRODUCTION

The fuzzy set introduced by L.A.Zadeh [16] in 1965 and the rough set introduced by Pawlak [12] in 1982 are generalizations of the classical set theory. Both these set theories are new mathematical tool to deal the uncertain, vague, imprecise and inexact data. In Zadeh fuzzy set theory, the degree of membership of elements of a set plays the key role, whereas in Pawlak rough set theory, the equivalence classes of a set are used to define the lower and upper approximation of a set.

Rosenfeld [13] applied the notion of fuzzy sets to groups and introduced the notion of fuzzy subgroups. After this paper, many researchers applied the theory of fuzzy sets to several algebraic concepts such as rings, fields, vector spaces, etc.

The notion of rough subgroups was introduced by Biswas and Nanda [1]. The concept of rough ideal in a semigroup was introduced by Kuroki in [11]. B.Davvaz [3], [2], [4] studied the roughness in many algebraic system such as rings, modules, n-ary systems, H_V -groups, etc. Osman Kazanci and B.Davvaz [10] introduced the rough prime and rough primary ideals in commutative rings and also discussed the roughness of fuzzy ideals in rings. The roughness of ideals in BCK algebras was considered by Y.B. Jun in [8]. In [14] the present authors have studied rough ideals in semirings.

In this paper, we introduce the concept of rough fuzzy ideal of a semiring. Also we study the notion of rough fuzzy prime ideal in a semiring.

2 CONGRUENCE IN SEMIRINGS

Definition 2.1. *A semiring is a nonempty set R on which operations of addition and multiplication have been defined such that the following conditions are satisfied.*

(i) $(R,+)$ *is a commutative monoid with identity element 0;*
(ii) $(R,\cdot)$ *is a monoid with identity element* 1_R *;*
(iii) Multiplication distributives over addition from either side;
(iv) $0r = 0 = r0$, *for all* $r \in R$.

Throughout this paper R denotes a semiring.

Definition 2.2. *[6] Let* κ *be an equivalence relation on* R, *then* κ *is called a congruence relation if* $(a,b)\in\kappa$ *implies* $(a+x,b+x)$, $(x+a,x+b)$, (ax,bx) and $(xa,xb)\in\kappa$ *for all* $x\in R$.

Theorem 2.3. *[6] Let* κ *be a congruence relation on* R, *then* $(a,b)\in\kappa$ *and* $(c,d)\in\kappa$ *imply* $(a+c,b+d)\in\kappa$ *and* $(ac,bd)\in\kappa$ *for all* $a,b,c,d\in R$.

Lemma 2.4. *[6] Let* κ *be a congruence relation on a semiring* R. *If* $(a,b)\in R$ *then*

(i) $\{a'+b' / a'\in[a]_\kappa, b'\in[b]_\kappa\}\subseteq[a+b]_\kappa$

(ii) $\{a'b' / a'\in[a]_\kappa, b'\in[b]_\kappa\}\subseteq[ab]_\kappa$

Definition 2.5. *A congruence relation* κ *on* R *is called complete if*

(i) $[a+b]_\kappa=\{a'+b' / a'\in[a]_\kappa, b'\in[b]_\kappa\}$ *and*

(ii) $[ab]_\kappa=\{a'b' / a'\in[a]_\kappa, b'\in[b]_\kappa\}$.

for all $a,b\in R$.

Definition 2.6. *A ideal* I *of a semiring* R *is a nonempty subset of* R *satisfying the following condition:*
(i) If $a,b\in I$ *then* $a+b\in I$.
(ii) If $a\in I$ *and* $r\in R$ *then* $ar, ra\in I$.

A ideal I of a semiring R defines an equivalence relation κ_I on R, called the Bourne relation, given by $r\kappa_I r'$ if and only if there exists elements a and a' of I satisfying $r+a=r'+a'$. The relation κ_I is an congruence relation on R [6], [7].

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We denote the set of all equivalence classes of elements of R under this relation by R/κ_I and we will denote the equivalence class of an element r of R by $[r]_{\kappa_I}$.

Throughout this paper κ_I denotes the Bourne congruence relation induced by an ideal I of a semiring R.

Definition 2.7. *An ideal I of a semiring R is called a κ-ideal if $r+a\in I$ implies $r\in I$ for each $r\in R$ and each $a\in I$.*

3 Lower And Upper Approximation of a Fuzzy Ideal In a Semiring

A mapping $\mu: R\to[0,1]$ is called a fuzzy subset of R.

A fuzzy subset μ of a semiring R is called a fuzzy ideal of R if it has the following properties:

(i) $\mu(x+y)\ge\mu(x)\wedge\mu(y)$

(ii) $\mu(xy)\ge\mu(x)\vee\mu(y)$

A fuzzy ideal μ of R is said to be normal if $\mu(0)=1$.

Definition 3.1. *A fuzzy ideal μ of a semiring R is said to be prime if μ is not a constant function and for any two fuzzy ideals σ and θ of $R, \sigma\circ\theta\subseteq\mu$ implies either $\sigma\subseteq\mu$* or *$\theta\subseteq\mu$.*

Definition 3.2. *Let κ_I be the Bourne congruence relation on R induced by I and μ be a fuzzy subset of R. Then we define the fuzzy sets $\underline{Apr}_{\kappa_I}(\mu)$* and *$\overline{Apr}_{\kappa_I}(\mu)$ as follows:*

$$\underline{Apr}_{\kappa_I}(\mu)(x)=\bigwedge_{a\in[x]_{\kappa_I}}\mu(a)$$

$$\overline{Apr}_{\kappa_I}(\mu)(x)=\bigvee_{a\in[x]_{\kappa_I}}\mu(a)$$

The fuzzy sets $\underline{Apr}_{\kappa_I}(\mu)$ and $\overline{Apr}_{\kappa_I}(\mu)$ are respectively called the κ_I-lower and κ_I-upper approximations of the fuzzy set μ. The pair $\left(\underline{Apr}_{\kappa_I}(\mu),\overline{Apr}_{\kappa_I}(\mu)\right)=Apr_{\kappa_I}(\mu)$ is called a rough fuzzy set with respect to κ_I if $\underline{Apr}_{\kappa_I}(\mu)\ne\overline{Apr}_{\kappa_I}(\mu)$.

Theorem 3.3. *For every approximation space (R,κ_I) and every fuzzy subsets μ,σ of R, we have:*

(1) $\underline{Apr}_{\kappa_I}(\mu)\subseteq\mu\subseteq\overline{Apr}_{\kappa_I}(\mu)$

(2) $\underline{Apr}_{\kappa_I}(\chi_\phi)=\chi_\phi$ *, where χ_ϕ is the characteristic function of empty set.*

(3) $\overline{Apr}_{\kappa_I}(\chi_R)=\chi_R$ *, where χ_R is the characteristic function of R.*

(4) *If $\mu\subseteq\sigma$ then $\underline{Apr}_{\kappa_I}(\mu)\subseteq\underline{Apr}_{\kappa_I}(\sigma)$*

(5) *If $\mu\subseteq\sigma$ then $\overline{Apr}_{\kappa_I}(\mu)\subseteq\overline{Apr}_{\kappa_I}(\sigma)$*

(6) $\underline{Apr}_{\kappa_I}\left(\underline{Apr}_{\kappa_I}(\mu)\right)=\underline{Apr}_{\kappa_I}(\mu)$

(7) $\overline{Apr}_{\kappa_I}\left(\overline{Apr}_{\kappa_I}(\mu)\right)=\overline{Apr}_{\kappa_I}(\mu)$

(8) $\overline{Apr}_{\kappa_I}\left(\underline{Apr}_{\kappa_I}(\mu)\right)=\underline{Apr}_{\kappa_I}(\mu)$

(9) $\underline{Apr}_{\kappa_I}\left(\overline{Apr}_{\kappa_I}(\mu)\right)=\overline{Apr}_{\kappa_I}(\mu)$

(10) $\underline{Apr}_{\kappa_I}(\mu\cap\sigma)=\underline{Apr}_{\kappa_I}(\mu)\cap\underline{Apr}_{\kappa_I}(\sigma)$

(11) $\overline{Apr}_{\kappa_I}(\mu\cap\sigma)\subseteq\overline{Apr}_{\kappa_I}(\mu)\cap\overline{Apr}_{\kappa_I}(\sigma)$

(12) $\underline{Apr}_{\kappa_I}(\mu\cup\sigma)\supseteq\underline{Apr}_{\kappa_I}(\mu)\cup\underline{Apr}_{\kappa_I}(\sigma)$

(13) $\overline{Apr}_{\kappa_I}(\mu\cup\sigma)=\overline{Apr}_{\kappa_I}(\mu)\cup\overline{Apr}_{\kappa_I}(\sigma)$

Proof. It is straightforward.

Theorem 3.4. *Let κ_I be the Bourne congruence relation on R induced by the ideal I of R. If μ is a fuzzy ideal of R, then $\overline{Apr}_{\kappa_I}(\mu)$ is a fuzzy ideal of R.*

Proof. We have,

$$\begin{aligned}\overline{Apr}_{\kappa_I}(\mu)(x+y) &= \bigvee_{a\in[x+y]\kappa_I}\mu(a)\\ &\ge \bigvee_{\substack{b\in[x]\kappa_I\\ c\in[y]\kappa_I}}\mu(b+c)\\ &\ge \bigvee_{\substack{b\in[x]\kappa_I\\ c\in[y]\kappa_I}}\left(\mu(b)\wedge\mu(c)\right)\\ &= \left(\bigvee_{b\in[x]\kappa_I}\mu(b)\right)\wedge\left(\bigvee_{c\in[y]\kappa_I}\mu(c)\right)\\ &= \overline{Apr}_{\kappa_I}(\mu)(x)\wedge\overline{Apr}_{\kappa_I}(\mu)(y)\end{aligned}$$

Hence, $\overline{Apr}_{\kappa_I}(\mu)(x+y)\ge\overline{Apr}_{\kappa_I}(\mu)(x)\wedge\overline{Apr}_{\kappa_I}(\mu)(y)$.

Also we have,

$$\begin{aligned}\overline{Apr}_{\kappa_I}(\mu)(xy) &= \bigvee_{a\in[xy]_{\kappa_I}}\mu(a)\\ &\ge \bigvee_{\substack{b\in[x]_{\kappa_I}\\ c\in[y]_{\kappa_I}}}\mu(bc)\\ &\ge \bigvee_{\substack{b\in[x]_{\kappa_I}\\ c\in[y]_{\kappa_I}}}\left(\mu(b)\vee\mu(c)\right)\\ &= \left(\bigvee_{b\in[x]_{\kappa_I}}\mu(b)\right)\vee\left(\bigvee_{c\in[y]_{\kappa_I}}\mu(c)\right)\\ &= \overline{Apr}_{\kappa_I}(\mu)(x)\vee\overline{Apr}_{\kappa_I}(\mu)(y)\end{aligned}$$

Hence, $\overline{Apr}_{\kappa_I}(\mu)(xy)\ge\overline{Apr}_{\kappa_I}(\mu)(x)\vee\overline{Apr}_{\kappa_I}(\mu)(y)$.

Therefore, $\overline{Apr}_{\kappa_I}(\mu)$ is a fuzzy ideal of R.

Theorem 3.5. *Let κ_I be the Bourne congruence relation on R induced by the ideal I of R. If μ is a fuzzy ideal of R, then $\underline{Apr}_{\kappa_I}(\mu)$ is a fuzzy ideal of R.*

Proof. We have,

$$\begin{aligned}\underline{Apr}_{\kappa_I}(\mu)(x+y) &= \bigwedge_{a\in[x+y]_{\kappa_I}} \mu(a)\\ &\geq \bigwedge_{\substack{b\in[x]_{\kappa_I}\\ c\in[y]_{\kappa_I}}} \mu(b+c)\\ &\geq \bigwedge_{\substack{b\in[x]_{\kappa_I}\\ c\in[y]_{\kappa_I}}} \left(\mu(b)\wedge\mu(c)\right)\\ &= \left(\bigwedge_{b\in[x]_{\kappa_I}} \mu(b)\right)\wedge\left(\bigwedge_{c\in[y]_{\kappa_I}} \mu(c)\right)\\ &= \underline{Apr}_{\kappa_I}(\mu)(x)\wedge\underline{Apr}_{\kappa_I}(\mu)(y)\end{aligned}$$

Hence, $\underline{Apr}_{\kappa_I}(\mu)(x+y)\geq \underline{Apr}_{\kappa_I}(\mu)(x)\wedge\underline{Apr}_{\kappa_I}(\mu)(y)$.

Also we have,

$$\begin{aligned}\underline{Apr}_{\kappa_I}(\mu)(xy) &= \bigwedge_{a\in[xy]_{\kappa_I}} \mu(a)\\ &\geq \bigwedge_{\substack{b\in[x]_{\kappa_I}\\ c\in[y]_{\kappa_I}}} \mu(bc)\\ &\geq \bigwedge_{\substack{b\in[x]_{\kappa_I}\\ c\in[y]_{\kappa_I}}} \left(\mu(b)\vee\mu(c)\right)\\ &= \left(\bigwedge_{b\in[x]_{\kappa_I}} \mu(b)\right)\vee\left(\bigwedge_{c\in[y]_{\kappa_I}} \mu(c)\right)\\ &= \underline{Apr}_{\kappa_I}(\mu)(x)\vee\underline{Apr}_{\kappa_I}(\mu)(y)\end{aligned}$$

Hence, $\underline{Apr}_{\kappa_I}(\mu)(xy)\geq \underline{Apr}_{\kappa_I}(\mu)(x)\vee\underline{Apr}_{\kappa_I}(\mu)(y)$.

Therefore, $\underline{Apr}_{\kappa_I}(\mu)$ is a fuzzy ideal of R.

Let μ be a fuzzy subset of R and $\left(\underline{Apr}_{\kappa_I}(\mu),\overline{Apr}_{\kappa_I}(\mu)\right)$ a rough fuzzy set. If $\underline{Apr}_{\kappa_I}(\mu)$ and $\overline{Apr}_{\kappa_I}(\mu)$ are fuzzy ideals of R, then $\left(\underline{Apr}_{\kappa_I}(\mu),\overline{Apr}_{\kappa_I}(\mu)\right)$ a rough fuzzy ideal of R.

Let μ and σ be two fuzzy subsets of R. The inclusion $\mu\subset\sigma$ is defined by $\mu(x)\leq\sigma(x)$ for all $x\in R$, and $\mu\cap\sigma$ is defined by

$$(\mu\cap\sigma)(x)=\mu(x)\wedge\sigma(x) \text{ for all } x\in R.$$

If μ and σ are fuzzy ideals of a semiring of R, then $\mu\cap\sigma$ is also a fuzzy ideal of R.

Corollary 3.6. *If μ is a fuzzy ideal of R then $\left(\underline{Apr}_{\kappa_I}(\mu),\overline{Apr}_{\kappa_I}(\mu)\right)$ is a rough fuzzy ideal of R.*

Corollary 3.7. *If μ and σ are fuzzy ideals of R then $\left(\underline{Apr}_{\kappa_I}(\mu\cap\sigma),\overline{Apr}_{\kappa_I}(\mu\cap\sigma)\right)$ is a rough fuzzy ideal of R.*

Let μ be a fuzzy subset of R. Then the sets $\mu_t=\{x\in R\,|\,\mu(x)\geq t\}$ and $\mu_t^s=\{x\in R\,|\,\mu(x)>t\}$ where $t\in[0,1]$ are called respectively, t-level subset and t-strong level subset of μ.

Theorem 3.8. *[7] Let μ be a fuzzy subset of R. Then μ is a fuzzy ideal of R iff μ_t and μ_t^s are, if they are non-empty, ideals of R for every $t\in[0,1]$.*

Theorem 3.9. *Let κ_I be a congruence relation on R. If μ is a fuzzy subset of R and $t\in[0,1]$, then*

(i) $\left(\underline{Apr}_{\kappa_I}(\mu)\right)_t=\underline{Apr}_{\kappa_I}(\mu_t)$

(ii) $\left(\overline{Apr}_{\kappa_I}(\mu)\right)_t^s=\overline{Apr}_{\kappa_I}(\mu_t^s)$

Proof.

(i) We have

$$\begin{aligned}x\in\left(\underline{Apr}_{\kappa_I}(\mu)\right)_t &\Leftrightarrow \underline{Apr}_{\kappa_I}(\mu)(x)\geq t\\ &\Leftrightarrow \bigwedge_{a\in[x]\kappa_I}\mu(a)\geq t\\ &\Leftrightarrow \mu(a)\geq t, \forall a\in[x]_{\kappa_I}\\ &\Leftrightarrow a\in\mu_t, \forall a\in[x]_{\kappa_I}\\ &\Leftrightarrow [x]_{\kappa_I}\subseteq\mu_t\\ &\Leftrightarrow x\in\underline{Apr}_{\kappa_I}(\mu_t)\end{aligned}$$

(ii) We have

$$\begin{aligned}x\in\left(\overline{Apr}_{\kappa_I}(\mu)\right)_t^s &\Leftrightarrow \overline{Apr}_{\kappa_I}(\mu)(x)>t\\ &\Leftrightarrow \bigvee_{a\in[x]_{\kappa_I}}\mu(a)>t\\ &\Leftrightarrow \mu(a)>t, \text{ for some } a\in[x]_{\kappa_I}\end{aligned}$$

$\Leftrightarrow a \in \mu_t^s$, for some $a \in [x]_{\kappa_I}$

$\Leftrightarrow [x]_{\kappa_I} \cap \mu_t^s \neq \phi$

$\Leftrightarrow x \in \overline{Apr}_{\kappa_I}(\mu_t^s)$

Lemma 3.10. *If μ is a normal fuzzy ideal of a semiring R and if $\underline{Apr}_{\kappa_I}(\mu) = \mu$ then $\chi_I \subseteq \mu$.*

Proof. Since $\underline{Apr}_{\kappa_I}(\mu) = \mu$, we have $\left(\underline{Apr}_{\kappa_I}(\mu)\right)(x) = \mu(x)$, $\forall x \in R$

That is, $\bigwedge_{a \in [x]_{\kappa_I}} \mu(a) = \mu(x), \forall x \in R$.

$\Rightarrow \mu(a) = \mu(x), \forall a \in [x]_{\kappa_I}$

In particular, $\mu(a) = \mu(0), \forall a \in [0]_{\kappa_I}$.

As μ is a normal fuzzy ideal of R, $\mu(0) = 1$.

Hence, $\mu(a) = \mu(0) = 1, \forall a \in [0]_{\kappa_I} \supseteq I$.

Thus, $\chi_I \subseteq \mu$.

The converse of the above leema is true if I is a κ-ideal of R.

Lemma 3.11. *If I is a κ–ideal of R such that $\chi_I \subseteq \mu$ then $\underline{Apr}_{\kappa_I}(\mu) = \mu$.*

Proof. Since $\chi_I \subseteq \mu$, we have $\mu(x) = 1, \forall x \in I$.

By Theorem 3.3, $\underline{Apr}_{\kappa_I}(\mu) \subseteq \mu$.

So, to complete the proof we need to show that $\mu \subseteq \underline{Apr}_{\kappa_I}(\mu)$.

i.e., we need to show that $\mu(x) \leq \underline{Apr}_{\kappa_I}(\mu)(x), \forall x \in R$.

i.e., we need to show that $\mu(x) \leq \mu(a), \forall a \in [x]_{\kappa_I}$.

Case (i) Let $x \notin I$ and $a \in [x]_{\kappa_I}$.

Then $a + i_1 = x + i_2$, for some $i_1, i_2 \in I$.

$\mu(a + i_1) = \mu(x + i_2) \geq \min\{\mu(x), \mu(i_2)\} = \mu(x)$.

In particular, $\mu(a) \geq \mu(x), \forall a \in [x]_{\kappa_I}$.

This implies that $\mu(x) \leq \left(\underline{Apr}_{\kappa_I}\mu\right)(x)$.

Case (ii) Let $x \in I$, then $\mu(x) = 1$.

We have $[x]_{\kappa_I} = [0]_{\kappa_I}$.

Since I is a κ–ideal, $[0]_{\kappa_I} = I$.

We have, $\left(\underline{Apr}_{\kappa_I}(\mu)\right)(x) = \bigwedge_{a \in [x]_{\kappa_I}} \mu(a) = \bigwedge_{a \in I} \mu(a) = 1 = \mu(x)$.

Hence in this case also, $\underline{Apr}_{\kappa_I}(\mu) = \mu$.

Combining Case (i) and (ii), we get $\mu \subseteq \underline{Apr}_{\kappa_I}(\mu)$

4 ROUGH FUZZY PRIME IDEALS

An ideal P of a semiring R is prime if and only if whenever $HK \subseteq P$, for ideals H and K of R, we must have either $H \subseteq P$ or $K \subseteq P$.

Theorem 4.1. *Let μ be a fuzzy prime ideal of a semiring R.*

(i) If κ_I is a complete congruence relation on R and $\underline{Apr}_{\kappa_I}(\mu) \neq \phi$, then μ is a lower rough fuzzy prime ideal of R.

(ii) If κ_I is a complete congruence relation on R then μ is a upper rough fuzzy prime ideal of R.

Proof.

(i) Since μ is a fuzzy prime ideal, $\mu_t(t \in [0,1])$ is, if it is non-empty, a prime ideal of R.

By Theorem 3.5. [14], we obtain that $\left(\underline{Apr}_{\kappa_I}(\mu_t)\right)$ if it is non-empty, is a prime ideal of R.

Hence by Theorem 3.9., $\left(\underline{Apr}_{\kappa_I}(\mu)\right)_t$ is a prime ideal of R and hence by Theorem 3.8., $\underline{Apr}_{\kappa_I}(\mu)$ is a fuzzy prime ideal of R.

(ii) It can be seen in a similar way.

Theorem 4.2. *Let κ_I be a complete congruence relation on a semiring R. Then μ is a lower (an upper) rough fuzzy prime ideal iff for $t \in [0,1] \mu_t, \mu_t^s$ are, if they are nonempty, lower [upper] rough prime ideals of R.*

Proof. It is straightforward.

Theorem 4.3. *[14] Let f be an onto homomorphism of a semiring R to a semiring R' and let κ_2 be a congruence relation on R' and A be a subset of R. Then*

(i) $\kappa_1 = \{(a,b) \in R \times R / (f(a), f(b) \in \kappa_2)\}$ *is a congruence relation on R.*

(ii) *If κ_2 is complete and f is one – to – one, then κ_1 is complete.*

(iii) $f\left(\overline{Apr}_{\kappa_1}(A)\right) = \overline{Apr}_{\kappa_2}(f(A))$

(iv) $f\left(\underline{Apr}_{\kappa_1}(A)\right) \subseteq \underline{Apr}_{\kappa_2}(f(A))$. *If f is one-to-one, then* $f\left(\underline{Apr}_{\kappa_1}(A)\right) = \underline{Apr}_{\kappa_2}(f(A))$.

Theorem 4.4. *Let f be a surjective homomorphism of a semiring R to a semiring R'. Let κ_2 be a complete congruence relation on R' and μ be a fuzzy subset of R.*

If $\kappa_1 = \{(x,y) \in R \times R / (f(x), f(y) \in \kappa_2)\}$, then

(i) $\overline{Apr}_{\kappa_1}(\mu)$ is a fuzzy ideal (fuzzy prime ideal) of R if and only if $\overline{Apr}_{\kappa_2}(f(\mu))$ is a fuzzy ideal (fuzzy prime ideal) of R'.

(ii) If f is one – to – one, then $\underline{Apr}_{\kappa_1}(\mu)$ is a fuzzy ideal (fuzzy prime ideal) of R if and only if $\underline{Apr}_{\kappa_1}(f(\mu))$ is a fuzzy ideal (fuzzy prime ideal) of R'.

Proof.

(i) By Theorem 3.5. [14] we obtain that $\overline{Apr}_{\kappa_1}(\mu)$ is a fuzzy ideal (fuzzy prime ideal) of R iff $\overline{Apr}_{\kappa_1}(\mu)_t^s$, if it is non-empty, an ideal (prime ideal) of R, for every $t \in [0,1]$.

We have by Theorem 3.9., $\overline{Apr}_{\kappa_1}(\mu)_t^s = \overline{Apr}_{\kappa_1}(\mu_t^s)$.

Thus we obtain that $\overline{Apr}_{\kappa_1}(\mu_t^s)$ is an ideal (prime ideal) of R iff $\overline{Apr}_{\kappa_2}(f(\mu_t^s))$ is an ideal (prime ideal) of R'.

Since $f(\mu_t^s) = (f(\mu))_t^s$, $\overline{Apr}_{\kappa_2}(f(\mu_t^s)) = \overline{Apr}_{\kappa_2}(f(\mu)_t^s)$

$= (\overline{Apr}_{\kappa_2} f(\mu)_t^s)$.

Therefore, $(\overline{Apr}_{\kappa_2} f(\mu))_t^s$ is an ideal (prime ideal) of R' for every $t \in [0,1]$.

Thus, $\overline{Apr}_{\kappa_2} f(\mu)$ is a fuzzy ideal (fuzzy prime ideal) of R'.

(ii) If f is one – to – one, then $f(\underline{Apr}_{\kappa_1}(A)) = \underline{Apr}_{\kappa_2}(f(A))$.

This proof is similar to that of (i).

5 Conclusion

The theory of semirings has wide applications in several areas such as optimization theory, discrete event dynamical systems, automata theory, formal language theory and parallel computing. The theory of fuzzy sets and rough sets also has many applications in the above areas. In this paper, we developed the concept of a rough fuzzy ideal of a semiring. We certainly hope that our work will be very useful both in the theoretical and application aspect. We also propose to work further on this area to bring out many more interesting properties of rough fuzzy ideals in semirings.

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